Comment on "Equilibrium Configurations and Energies of the Rotating Elastic Cable in Space"

J. Pike*
Holly Cottage, Chawston, Bedford, United Kingdom

IN Ref. 1, it is suggested that a thin elastic rod rotating in space can have stable equilibrium configurations that contain intersections or loops. The conditions for stability, which are neglected by Wang, are given here together with a derivation of a simplified version of the energy equation. Unfortunately, these results cannot be used to demonstrate the stability of Wang's looped configurations[1] because they are found to be unstable.

For the rod to be stable, the configuration must be at a local energy minimum for any small perturbation of the shape that maintains the angular momentum. The angular momentum is given in a nondimensional form by

$$A = \frac{\text{angular momentum}}{l\rho^{1/2}EI^{1/2}} = J^2\int_0^1 y^2\,\mathrm{d}s \tag{1}$$

where l is the rod length, ρ the rod mass per unit length, and EI the rod stiffness. The nondimensional parameter J is $l\rho^{1/4}\theta^{1/2}/EI^{1/4}$, which is misquoted by Wang in Eq. (7) of Ref. 1. The coordinate along the rod and the distance of the rod from the axis of rotation are s and $y(s)$, respectively. They are nondimensionalized with respect to the rod length.

The energy is given by Eq. (16) of Wang's paper to be

$$\epsilon = \frac{2l\,\text{energy}}{EI} = J^4\int_0^1 y^2\,\mathrm{d}s + \int_0^1\left(\frac{\mathrm{d}\theta}{\mathrm{d}s}\right)^2\mathrm{d}s \tag{2}$$

where $\sin\theta = -\mathrm{d}y/\mathrm{d}s$. This equation gives the rod energy for any instantaneously stationary shape. For rods in equilibrium,[1,2] the energy equation can be simplified by substituting for θ from the local equilbrium conditions on the rod. We write

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}s}\right)^2 = 2\int_0^s \frac{\mathrm{d}\theta}{\mathrm{d}s}\frac{\mathrm{d}^2\theta}{\mathrm{d}s^2}\,\mathrm{d}s = -J^4(y^2 - y_{\max} + 2u\sin\theta) \tag{3}$$

where

$$u(s) = \int_0^s y\,\mathrm{d}s$$

For the rotation axis to be in equilibrium, we must have $u(1) = 0$, and hence ϵ can be reduced to

$$\epsilon = J^4 y_{\max}^2 - 2J^2 A \tag{4}$$

For stability, Eq. (4) with A constant must represent a local minimum among the stationary rod shapes whose energy is given by Eq. (2). The instability argument that Wang uses based on the increase of angular velocity with $y_{\max}$ does not appear to be valid in this context. Note that Eq. (4) enables values of the angular momentum for equilibrium configurations to be obtained from the values of ϵ and $y_{\max}$ given by Wang.

All of the equilibrium configurations investigated by Wang lie in a plane containing the axis of rotation. At the crossover point, the rod center will be displaced slightly from this plane by the thickness of the rod. The rod ends, when applying the force to maintain the loop curvature, will also apply a small moment to the loop. This moment is orientated so as to seek to unwind the loop. For a rod whose stiffness is homogeneous, the loop can apply no counterbalancing moment, and the loop will unwind. The looped configurations found by Wang are thus unstable, unless the stiffness of the rod varies with the direction in which the rod is bent. However, for this restricted class of rods, the looped configurations in a plane perpendicular to the axis of rotation, which are not investigated by Wang, would appear to be more likely equilibrium shapes for the rod to attain.

Received May 22, 1987.

*Member AIAA.

References

[1]Wang, C. Y., "Equilibrium Configurations and Energies of the Rotating Elastic Cable in Space," *AIAA Journal*, Vol. 24, Dec. 1986, pp. 2010–2013.

[2]Wang, C. Y., "Rotation of a Free Elastic Rod," *Journal of Applied Mechanics*, Vol. 49, March 1982, pp. 225–227.

Reply by Author to J. Pike

C. Y. Wang*
Michigan State University, East Lansing, Michigan

THE comment and corresponding reply can be separated into the following items:

1) There is a typographical error in Eq. (7) of Ref. 1. The author agrees that a minus sign in the exponent of EI is missing. The correct form is

$$J \equiv l\rho^{1/4}\Omega^{1/2}(EI)^{-1/4}$$

Incidentally, the correction suggested by Pike, $l\rho^{1/4}\theta^{1/2}/EI^{1/4}$, is still erroneous.

2) Pike stated, "The instability argument that Wang uses based on the increase of angular velocity with $y_{\max}$ does not appear to be valid. . . ." Pike misread the paper. The instability criterion in the last paragraph of Ref. 1 is clearly $\mathrm{d}\epsilon/\mathrm{d}J < 0$ instead. Pike suggests a stability criterion that total energy ϵ must be a local minimum among the stationary shapes with angular momentum unchanged. However, angular momentum is not independent. It changes with J and ϵ for stationary equilibrium shapes. See Fig. 10 of Ref. 2, where the angular momentum is computed. Perhaps a nonequilibrium dynamical approach may be more appropriate.

3) Pike gives arguments that the crossover configurations are unstable due to the forces at the contact point for finite thickness. The author disagrees. Satellite antennas that could loop have lengths thousands of times greater than their thickness. Thus, the effects of reactive forces at the contact point are as insignificant as imperfection perturbations, such as natural curvature, inhomogeneity, etc. A better question is whether all rotating cables, looped or not, are stable to small out-of-plane disturbances. Without going through the necessary computations, the answer is probably yes. The small out-of-plane disturbances create a twist that is resisted by torsional rigidity.

References

[1]Wang, C. Y., "Equilibrium Configurations and Energies of the Rotating Elastic Cable in Space," *AIAA Journal*, Vol. 24, Dec. 1986, pp. 2010–2013.

[2]Wang, C. Y., "Free Rotation of an Elastic Rod with an End Mass," *Journal of Applied Mechanics*, Vol. 53, 1986, pp. 864–868.

Submitted Feb. 16, 1988.

*Professor, Department of Mathematics and Mechanical Engineering.